Worksheet on chapter 5 and 6

1) Differentiate: $\cos x^{3} \cdot \sin ^{2} x^{5} \quad$ w.r.t.x
2) Find: $\frac{d y}{d x}$ if $\mathrm{ax}^{2}+2 \mathrm{~h} x y+\mathrm{by}^{2}=0$
3) Find: $\frac{d y}{d x}$ if $y=\tan ^{-1}\left(\frac{3 x-(x) 3}{1-3 x}{ }^{2}\right)$
4) Differentiate: $\frac{\sqrt{\left.\llbracket(x-3) x^{2}+4\right)}}{\sqrt{3 \mathrm{x}^{2}+4 \mathrm{x}+5}}$ w.r.t.x
5) Find: $\frac{d y}{d x}$ if $y^{x}+x^{y}+x^{x}=a^{b}$
6) Differentiate: $x^{\cos x}+(\cos x)^{\sin x}$ w.r.t.x
7) If $\mathrm{x}=\sqrt{ } a^{\sin ^{-1}} \mathrm{t} ; \mathrm{y}=\sqrt{ } a^{\cos ^{-1}} \mathrm{t}$ show that $\frac{d y}{d x}=\frac{-y}{x}$
8) If $x=\cos \theta-\cos 2 \theta ; y=\sin \theta-\sin 2 \theta$ find $\frac{d y}{d x}$
9) If $y=3 \cos (10 g x)+4 \sin (10 g x)$ show that $x^{2} y_{2}+x y_{1}+y=0$
10) If $y=\left(\tan ^{-1} x\right)^{2}$ show that $\left(x^{2}+1\right)^{2} y_{2}+2 x\left(x^{2}+1\right) y_{1}=2$
11) Differentiate: $\cos ^{2} x$ w.r.t. $e^{\sin x}$
12) Differentiate: $\cot ^{-1}\left[\frac{\sqrt{1+\sin x^{1}}}{\sqrt{1+\sin x^{1}}} \pm \frac{\sqrt{1-\sin x^{1}}}{\sqrt{1-\sin x^{1}}}\right]$ w.r.t.x
13) If $x \sqrt{1+y^{1}}+y \sqrt{1+x^{1}}=0$ prove that $\frac{d y}{d x}=\frac{-1}{(1+x)^{2}}$
14) Find the interval in which the fuction $f$ given by $f(x)=\sin x+\cos x ; 0 \leq x \leq 2 \pi$ is strictly increasing or strictly decreasing.
15) A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is tan (0.5). Water is poured into it at a constant rate of 5 cubic meters per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m .
16) Show that the function $f$ given by $f 9 x)=\tan ^{-1}(\sin x+\cos x) ; x \geq 0$ always an strictly increasing function in $\left(0, \frac{\pi}{4}\right)$
17) A point on the hypotenuse of a triangle is at distance ' $a$ ' and ' $b$ ' from the sides of the triangle. Show that the maximum length of the hypotenuse is $\left.a^{2 / 3}+b^{2 / 3}\right)^{3 / 2}$
18) Using differentiation find the approximate value of $(33)^{-1 / 5}$
19) Find the equation of the normal to the curve $x^{2}=4 y$ which passes through the point $(1,2)$
20) Find the equation of tangent to the curve $y=\cos (x+y) ;-2 \pi \leq x \leq 2 \pi$ that are parallel to $x+2 y=0$.
